Partiell rekursive Funktionen – Beispiele

Definition:

$$g : \mathbb{N}^{k+1} \longrightarrow \mathbb{N}$$

$$\mathsf{MIN} : (\mathbb{N}^{k+1} \longrightarrow \mathbb{N}) \longrightarrow \mathbb{N}^k \longrightarrow \mathbb{N} \quad \mathsf{MIN}(g) : \mathbb{N}^k \longrightarrow \mathbb{N}$$

$$\mathsf{MIN}(g)(x_1, \ldots, x_k) = \mu t(g(x_1, \ldots, x_k, t) = 0)$$

$$\mu t(g(x_1, \ldots, x_k, t) = 0) = \begin{cases} z & \min \left\{ \begin{array}{l} t \in \mathbb{N} \mid g(x_1, \ldots, x_k, t) = 0 \\ \wedge \forall i < t : g(x_1, \ldots, x_k, i) \in \mathbb{N} \end{array} \right\} = z \in \mathbb{N} \\ \text{undef} & \text{sonst} \end{cases}$$

Beispiel aus der Vorlesung:

$$g = s \quad (k = 0) \quad \mathsf{MIN}(g) = \mu t(s(t) = 0) = \text{undef}$$

weitere Beispiele

$$g(x) = x - 1 \quad (k = 0) \quad \mathsf{MIN}(g) = \mu t(x - 1 = 0) = 0$$

$$g = + \quad (k = 1) \quad \mathsf{MIN}(g)(x) = \mu t(x + t = 0) = \begin{cases} 0 & x = 0 \\ \text{undef} & \text{sonst} \end{cases}$$

$$g = - \quad (k = 1) \quad \mathsf{MIN}(g)(x) = \mu t(x - t = 0) = x$$

$$g = \cdot \quad (k = 1) \quad \mathsf{MIN}(g)(x) = \mu t(x \cdot t = 0) = 0$$

$$g = \mod \quad (k = 1) \quad \mathsf{MIN}(g) = \mu t(x \mod t = 0) = \begin{cases} 0 & x = 0 \\ 1 & x > 0 \end{cases}$$

geschachtelt

$$f = R(s, \mathsf{MIN}(I_1^4))$$

$$\begin{aligned} f(x, 0) &= s(x) = x + 1 \\ f(x, y + 1) &= \mathsf{MIN}(I_1^4)(x, y, f(x, y)) = \mu t(I_1^4(x, y, f(x, y), t) = 0) = \begin{cases} 0 & \text{falls } x = 0 \\ \text{undef} & \text{sonst} \end{cases} \\ f(x, y) &= \begin{cases} x + 1 & \text{falls } y = 0 \\ 0 & \text{falls } x = 0, y > 0 \\ \text{undef} & \text{sonst} \end{cases} \end{aligned}$$

$$f' = \mathsf{MIN}(f) = \mathsf{MIN}(R(s, \mathsf{MIN}(I_1^4)))$$

$$f'(x) = \mu t(f(x, t) = 0)(x) = \begin{cases} 1 & \text{falls } x = 0 \\ \text{undef} & \text{sonst} \end{cases}$$